sets of conditions in enclosed containers of power plants, while they also support the results obtained in [8] at elevated pressures.

NOTATION

$\overline{P} = P/P_s$, P, and P_s, relative pressure, present pressure, and saturated vapor pressure, respectively; $Fo_D = D\tau/l^2$, diffusion Fourier number; D, diffusion coefficient; l, height of gas above the liquid surface; τ, time.

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AN EXPRESSION FOR THE GROWTH MODULUS OF VAPOR BUBBLES AT THE BOILING POINT

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An expression is proposed for the growth modulus of a vapor bubble when its radius varies in an arbitrary manner with time.

1. A considerable number of theoretical papers have been devoted to the growth laws of vapor bubbles at the boiling point [1-8]. All lead to the equation

$$R = \beta(0.5)\,\tau^{0.5}, \tag{1}$$

which is true for most of the period of growth of the vapor bubble on the heating surface. If we exclude cases of extremely low pressures, Eq. (1) may be treated as an approximate theoretical law for the growth of vapor bubbles at the boiling point.

The growth modulus $\beta(0.5)$ of a vapor bubble is, in general, expressed by the following equation:

$$\beta(0.5) = c_\beta \left(\frac{\lambda \Delta T}{L\rho'' a}\right)^{n_\beta} a^{0.5} = c_\beta\, \mathrm{Ja}^{n_\beta} a^{0.5}, \tag{2}$$

where n_β lies in the range $0.5 \leqslant n_\beta \leqslant 1$, while c_β = const (in some papers c_β is defined as a function of the physical properties of the heater and test liquid [5] but this is not particularly important for subsequent analysis).

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At sufficiently high pressures we may put $n_\beta = 0.5$ and at sufficiently low pressures $n_\beta = 1$ [4]. The boundary between "high" and "low" pressures in this context is clearly defined by the Jacob number Ja, which is of the order of 10 [4, 8, 9].

2. Experimental investigations show that in many cases the growth law of the bubble differs from Eq. (1), and a more accurate approximation for the average time dependence of the radius of the bubble may be obtained by means of the equation

$$R = \beta(n)\tau^n, \tag{3}$$

where n varies over an extremely wide range such as $0.25 < n < 0.7$ [7-14].

A study of the deviation of the experimental relationship (3) from the theoretical equation (1) is especially important if the power index n is a function of parameters having a major effect on the boiling process (for example, if n is a function of pressure or gravitational acceleration). If we know how the growth of the bubble varies in relation to these parameters, we may introduce appropriate corrections into the estimates of the internal and integrated boiling characteristics, for example, the separation dimensions and separation frequency of the vapor bubbles [15].

The question as to the dependence of the index n in (3) on various parameters of state has hardly been studied at all. Certain qualitative conclusions may be drawn if we analyze existing experimental data. Thus the power index n in (3) evidently falls with increasing pressure [9] and diminishing gravitational acceleration [10, 11]. In the boiling of water on Teflon-coated heaters, $n = 0.25$ over an extremely wide range of pressures and thermal fluxes [12].

In this paper we shall estimate the growth modulus $\beta(n)$ for an arbitrary, known value of n, without considering the reasons for the deviation of (3) from the theoretical equation (1).

3. If Eq. (2) for $\beta(0.5)$ is correct, the structure of the general expression must be such that $\beta(n) \equiv \beta(0.5)$ when $n = 0.5$.

The simplest expression for $\beta(n)$ has the form

$$\beta(n) = \beta(0.5)\, a^{n-0.5}\, l^{1-2n}. \tag{4}$$

It is reasonable to consider that the quantity l, having the dimensions of length, is equal or proportional to the thickness of the superheated layer close to the heater, which provides the heat used in evaporating the liquid into the bubble.

The validity of this assumption may be verified by considering a specific model leading to a bubble growth law differing from (1).

Let us consider a spherical bubble growing on the surface of the heater (Fig. 1). Let heat pass to the bubble solely through that part of its surface limited by the thickness of the superheated layer h (as in the Labuntsov model [4]); the supply of heat to the bubble does not arrive directly from the heater by conduction, but arises from the enthalpy of the superheated liquid (as in the Plesset–Zwick model [1]) in the region of the superheated layer between the heater and the bubble.

For simplicity, we shall consider the temperature of the superheated layer as being equal to the temperature of the heater T_H, and the temperature of the surface of the bubble and the liquid outside the layer as being equal to the saturation temperature T_s. Using the simplified mode of calculation proposed in [1], we then obtain the following relationship from the thermal flux equations (including the flux used in evaporation and that conveyed to the bubble by conduction):

$$4\pi R^2 L\rho'' \frac{dR}{d\tau} = \lambda \frac{\Delta T}{(a\tau)^{0.5}} \pi h(4R - h). \tag{5}$$

Considering $h \ll R$ and integrating Eq. (5), we obtain

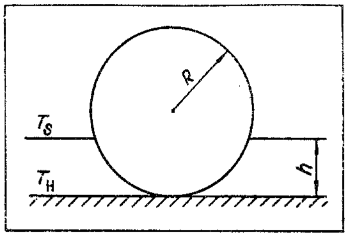

Fig. 1. Spherical bubble growing on the heater surface.

TABLE 1. Calculation of Bubble-Growth Moduli for the Boiling of Oxygen and Nitrogen

Substance	Oxygen					Nitrogen			
p, bar	0,1	0,25	0,42	0,7	1,0	0,22	0,45	0,7	1,0
n	0,53	0,72	0,47	0,48	0,46	0,66	0,55	0,49	0,51
$\beta_{cal}(n)$, cm/sec^n	2,52	3,11	0,50	0,48	0,30	1,68	0,51	0,34	0,22
β_{exp}/β_{cal}	1,39	1,70	1,65	1,41	1,35	1,49	1,76	1,34	2,0

TABLE 2. Calculation of Bubble-Growth Moduli for the Boiling of Nitrogen

References	[14]			[13]	
ΔT, deg	8,2	2,15	6,1	1,3	1,84
n	0,45	0,38	0,43	0,40	0,45
$\beta_{exp}(n)$, cm/sec^n	0,40	0,16	0,45	0,116	0,166
$\beta_{cal}(n)$, cm/sec^n	0,43	0,078	0,29	0,061	0,110
β_{exp}/β_{cal}	0,92	2,06	1,56	1,91	1,5

$$R = 2\left(\frac{\lambda \Delta T}{L\rho''}\right)^{0.5} \frac{h^{0.5}}{a^{0.25}} \tau^{0,25} = \beta\,(0.25)\,\tau^{0.25}. \quad (6)$$

The values of $\beta(0.25)$ from (6) and (4) coincide, apart from a constant factor, if we consider that $l = h$ and $\beta(0.5)$ in Eq. (4) is given by the Labuntsov equation [4] $\beta(0.5) \sim (\lambda\Delta T/L\rho'')^{0.5}$. In deriving (5) and (6) we assumed that the growing bubble was of spherical shape, touching the heater at a single point (this is equivalent to the assumption of good wetting properties). This limitation is not very severe, since it only affects the value of the constant coefficient in the bubble growth modulus.

As already indicated, the radius of the growing bubbles associated with the boiling of water on surfaces with a low thermal conductivity varies in accordance with an $R \sim \tau^{0.25}$ law [12].

4. The question as to the specific expression for l remains open. It is reasonable to assume that the quantity l is associated with the dimensions R_c of the vaporization centers (microscopic indentations) which are initiated for a specific thickness of the superheated liquid at the heater l and a specific temperature head ΔT. This proposition has been repeatedly advanced by various authors [16, 17]. The quantity R_c may, in turn, be regarded as a function of the critical radius of the vapor nucleus [17],

$$R_* = \frac{2\,T_s\sigma}{L\rho''\Delta T}\ . \quad (7)$$

Assuming a very simple linear relationship between these quantities, $l = AR_*$, and making use of (2) and (4), we obtain the following general expression for the growth modulus of the vapor bubble:

$$\beta(n) = c_\beta (2A)^{1-2n} \left(\frac{\lambda \Delta T}{L\rho'' a}\right)^{n_\beta} a^n \left(\frac{T_s \sigma}{L\rho'' \Delta T}\right)^{1-2n}, \tag{8}$$

where $n_\beta = 0.5$ under the conditions in which the Labuntsov formula for $\beta(0.5)$ is valid, and $n_\beta = 1$ when the Plesset–Zwick formula applies. It is easy to see that when $n = 0.25$ the quantity $\beta(n) = \beta(0.25)$ does not depend on ΔT and hence on the thermal flux density; this agrees with the experimental results obtained in [12].

The value of the constant A may be determined more exactly by reference to experimental data. To a first approximation we may take $A \approx 10$, which is supported by an analysis of the experimental data of [9] presented in Table 1.

The deviation of the experimental values of the growth modulus β_{exp} from the calculated values β_{cal} is due to the "theoretical" value of the coefficient c_β, which is too low for the experimental conditions in question. We see from Table 1 that, even on using the theoretical formula of Plesset and Zwick [1] for the calculation [when the bubbles grow in accordance with the "theoretical" equation (1), i.e., when the n in (3) is 0.5], the ratio β_{exp}/β_{cal} differs considerably from unity. Deviations of the same order are observed on analyzing the bubble-growth curves associated with the boiling of nitrogen under a pressure of $p \approx 1$ bar given in [13, 14] (Table 2). If we take the value of c_β as equal to 3.06, then for the whole set of experimental data we obtain an average value of β_{exp}/β_{cal} with a comparatively slight mean-square error of ±18%.

NOTATION

a, thermal diffusivity; A, c_β, n_β, n, constants; h, thickness of the superheated layer at the heater; l, characteristic length; L, latent heat of vaporization; R, radius of bubble; R_*, critical radius of vapor nucleus; T_s, saturation temperature; ΔT, temperature head; p, pressure; β, $\beta(n)$, $\beta(0.5)$, growth modulus of vapor bubble; τ, time; ρ'', vapor density; λ, thermal conductivity; σ, surface tension; $Ja = \lambda\Delta T/L\rho'' a$, Jacob number. Indices: β, constants in the expression for the growth modulus; cal, calculated value; exp, experimental value.

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ANALOGIES BETWEEN HEAT AND MOMENTUM TRANSFER IN A FORCED LIQUID FLOW WITH SURFACE BOILING IN A CHANNEL

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Consideration is made of the characteristics of heat- and momentum-transfer phenomena in a forced liquid flow under conditions of surface boiling in a channel.

There are very few analytical studies of the hydrodynamics of a forced liquid flow under conditions of surface boiling in a channel. The attempt to apply a Reynolds analogy to describe this process [1] is well known, and the analogy yielded good results for the case of a Prandtl number close to unity. In the literature there are also generalizations of experimental data on the basis of similarity theory [2-5] which satisfactorily describe the experimental results, as a rule, that are characterized by Prandtl numbers close to unity. The subject of the present study is the development of the relationship between friction and heat exchange for surface boiling in a forced flow of nonmetallic liquid for a relatively broad range of the Prandtl number.

We study the forced liquid flow in a channel under conditions of surface boiling for underheatings sufficiently deep to consider the steam content of the flow to be equal to zero. We assume that the momentum transfer onto the channel wall is conditioned both as a usual eddy viscosity characteristic for a single-phase flow and a specific mechanism for the intermixing of the liquid mass at the underheating surface characteristic of the boiling process. We assume that in the general case the shearing stress corresponding to the mechanism of the momentum transfer interacts with the so-called heat flow of boiling q_{bp} that is equal to the difference between the total heat flow on the wall and the convective heat flow of a single-phase liquid under the same dynamic and temperature conditions. For these assumptions the shearing stress on the wall is

$$\tau = \tau_0 + \tau_{bp}. \tag{1}$$

It is evident that the calculation of τ_{bp} should be based on the determined representations of the character of the process of surface boiling. We assume that the basic factor conditioning both momentum and heat transfer under conditions of surface boiling is the intermixing of the mass of the liquid phase of the heat-transfer agent that is produced by the boiling process. The steam bubbles in the initial period on the wall at the highest intensity of their mixing action are considered immovable with respect to the flow core. We can also consider that the microcirculation of the liquid arising in the region of the growing bubble is symmetric with respect to the bubble. Consequently, under these assumptions the liquid volume adjoining the bubble on the average will be immovable with respect to the channel wall.

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